SUPPLEMENTARY MATERIAL

INDIVIDUAL VARIABILITY IN TREE ALLOMETRY DETERMINES LIGHT RESOURCE ALLOCATION IN FOREST ECOSYSTEMS – A HIERARCHICAL BAYESIAN APPROACH.

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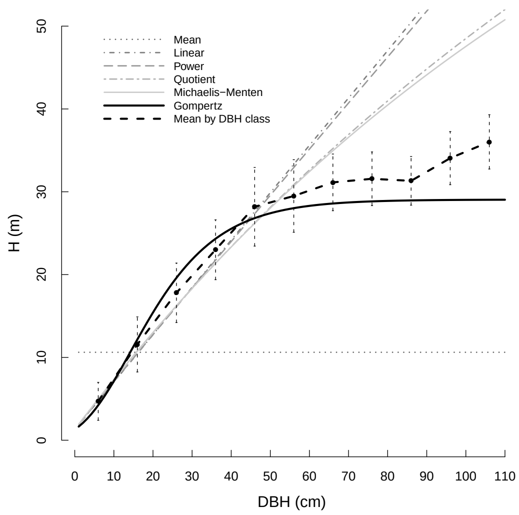
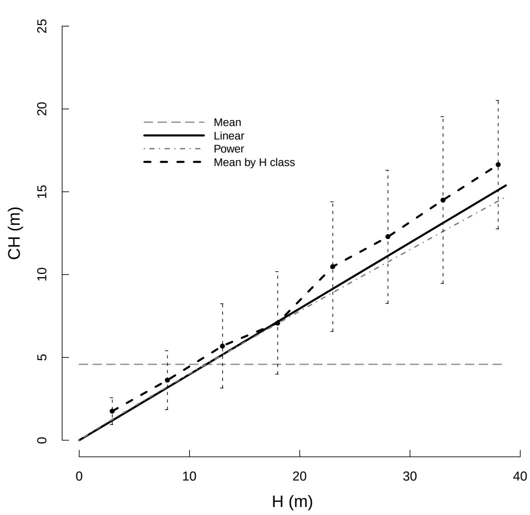
Site number	Site name	Country	Alps region	Elevation (m)	Latitude	Longitude	Surface (ha)	Number of trees (first census)	Species (% of stems at first census)			First census	Second census
									Abies alba	*Picea abies*	Others		
1	Luan	Switzerland	Canton of Vaud	1442	46° 21' 45” N	6° 58' 16” E	1.00	339	24	22	54	2004	—
2	Miroir1	France	Tarentaise	1357	45° 36' 18" N	6° 53' 07" E	0.25	375	95	5	0	1994	2006
3	Miroir3	France	Tarentaise	1377	45° 36' 19" N	6° 53' 09" E	0.25	319	91	9	0	1994	2006
4	Premol	France	Belledone	1434	45° 06' 41" N	5° 51' 26" E	0.80	503	34	45	20	2005	—
5	Queige	France	Beaufortain	1358	45° 41' 57" N	6° 27' 30" E	0.50	285	51	49	0	2002	—
6	Sixt	France	Haut Giffre	1520	46° 01' 16" N	6° 48' 51" E	0.25	608	1	95	5	1994	2006
7	SteFoy	France	Tarentaise	1642	45° 33' 08" N	6° 54' 23" E	0.25	219	0	99	1	1994	2006
8	StRhemy	Italy	Aosta Valley	1874	45° 50' 16” N	7° 11' 18” E	0.30	96	0	91	9	2003	—
9	Teppas	Italy	Aosta Valley	1720	45° 02' 36" N	6° 40' 30" E	2.00	939	73	21	6	1998	—

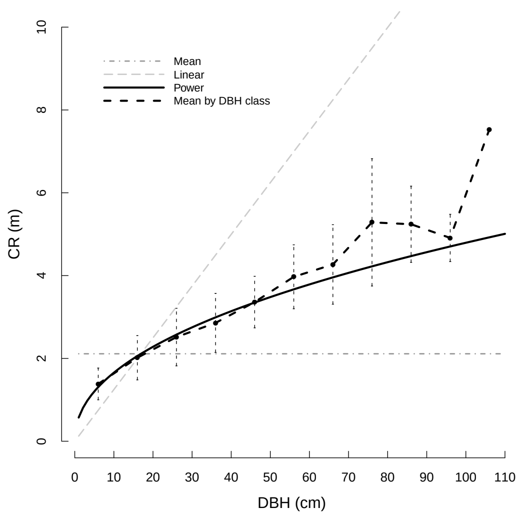
Appendix S1: **Plot characteristics.** Trees were measured on nine different plots ranging in size from 0.25 ha to 1 ha. Six plots were located in the French Alps, two in the Italian Alps and one in the Swiss Alps. Stands are uneaven-aged. *Abies alba* Mill. (Silver Fir) and *Picea abies* (L.) Karst. (Norway spruce) are the dominant species. All sites are situated at the mountain-belt elevation from 800 to 1800 m.



(a)



(b)



(c)

Appendix S2: **Graphical superposition of calibrated mathematical functions with points representing the mean of the response by covariate class**. Allometries are: (a) height as a function of DBH, (b) crown height as a function of height and (c) crown radius as a function of DBH. Some parametric functions may be too much constrained by an unbalanced data-set, where the number of smaller trees is much more important than the number of bigger trees. Here we show that the graphical superposition of the mathematical function selected and the mean by DBH class (or H class) was good and that selected models were not biased because of an unbalanced data-set.

(a) H-DBH

Model description	Model number	Mathematical function	Parameters	Effects (Y: yes, n: no)	Posterior mean of deviance	pD	DIC
				Covariate DBH			
Mean model	H1	$y = m$	m	n	4012.59	1.99	4014.58
Linear model	H2	$y=1.3+ax$	a	Y	276.47	1.94	278.41
Power model	H3	$y=1.3+ax^b$	a,b	Y	271.90	3.25	275.15
Monod model	H4	$y=1.3+\frac{ax}{b+x}$	a,b	Y	176.10	2.97	179.07
Michaelis-Menten model	H5	$y=1.3+K\,e^{\left(-\frac{c}{K}x\right)}$	K,c	Y	168.09	3.18	171.26
Gompertz model	H6	$y = K\,e^{\left(-\log\left(\frac{K}{1.3}\right)e^{(-rx)}\right)}$	K,r	Y	-198.82	3.05	-195.78

(b) CH-H

Model description	Model number	Mathematical function	Parameters	Effects (Y: yes, n: no)	Posterior mean of deviance	pD	DIC
				Covariate H			
Mean model	CH1	$y = m$	m	n	2868.67	1.98	2870.65
Power model	CH2	$y=ax^b$	a,b	Y	1521.62	2.90	1524.52
Linear model	CH3	$y=\frac{1}{1+K}x$	K	Y	1529.80	2.01	1531.81

(c) CR-DBH

Model description	Model number	Mathematical function	Parameters	Effects (Y: yes, n: no)	Posterior mean of deviance	pD	DIC
				Covariate DBH			
Mean model	CR1	$y = m$	m	n	1685.17	1.99	1687.15
Linear model	CR2	$y=ax$	a	Y	1862.47	2.01	1864.48
Power model	CR3	$y=Kx^b$	K,b	Y	301.95	2.96	304.91

Appendix S3: **Model comparison for the three allometric relations**. Allometries are (a) height as a function of DBH, (b) crown height as a function of height and (c) crown radius as a function of DBH. The lower the DIC, the best the model. A difference of more than 10 in the DIC rules out the model with the higher DIC. For equivalent DIC, we selected the model with the lower deviance. If the deviance difference was inferior to 10, we applied the parsimonious principle selecting the model with fewer parameters (with the lowest pD).

Appendix S4: Measurement errors

Model for measurement errors

Indexes and notations

i: Index of the tree.
t: Index of the measuring team.
T: Number of measurements for each tree ($T = 3$).
I: Number of trees in the measurement error protocol ($I = 50$).
z_{it}: Measurement t of variable for tree i. z can be DBH, height, crown height or crown radius.
$\boldsymbol{Z}$: Vector of observed values z_{it}.
$z_{i,0}$: Latent variable ("true value") z for tree i.
$\boldsymbol{Z_0}$: Vector of "true values" $z_{i,0}$.
σ_z^2: Variance for measurement errors.
N: Normal distribution.
LN: Log-normal distribution.
IG: Inverse-gamma distribution.

Bayes formula

$p(\text{parameter}|\text{data}, \text{model}) \propto \text{Likelihood} \times \text{Prior}$

Likelihood

The likelihood is defined as the probability of observing the data under the assumption that the model is true:
$p(\boldsymbol{Z}|\boldsymbol{Z_0}, \sigma_z^2) = \prod_{t=1}^{T} \prod_{i=1}^{I} LN(z_{it}|\log(z_{i,o}), \sigma_z^2)$

Priors

$p(\log(z_{i,0})) = N(\log(z_{i,0})|u_i, v_i)$, with $u_i = 0$ and $v_i = 1.0 \times 10^6$
$p(\sigma_z^2) = IG(\sigma_z^2|s_1, s_2)$, with $s_1 = 1.0 \times 10^{-3}$ and $s_2 = 1.0 \times 10^{-3}$

Joint posterior

$p(\boldsymbol{Z_0}, \sigma_z^2|\boldsymbol{Z}, \text{priors}) \propto \prod_{t=1}^{T} \prod_{i=1}^{I} p(z_{it}|z_{i,o}, \sigma_z^2)p(z_{i,0})p(\sigma_z^2)$
$p(\boldsymbol{Z_0}, \sigma_z^2|\boldsymbol{Z}, \text{priors}) \propto \prod_{t=1}^{T} \prod_{i=1}^{I} LN(z_{it}|\log(z_{i,o}), \sigma_z^2)N(\log(z_{i,0})|u_i, v_i)IG(\sigma_z^2|s_1, s_2)$

Conditional posterior for parameter σ_z^2

$p(\sigma_z^2|\boldsymbol{Z}, \boldsymbol{Z_0}, \text{priors}) \propto \prod_{t=1}^{T} \prod_{i=1}^{I} LN(z_{it}|\log(z_{i,o}), \sigma_z^2)IG(\sigma_z^2|s_1, s_2)$

Measurement error results

MCMC provided 1000 estimates for σ_z^2. The mean and standard variation for σ_z^2 were calculated for each dendrometric variable (Tab. S4). We were able to estimate the precision of our measurement as a percentage (Tab. S4) because we considered multiplicative errors: $z_{it} = z_{i,0}\exp(\epsilon_{it})$. For a 95% confidence interval:

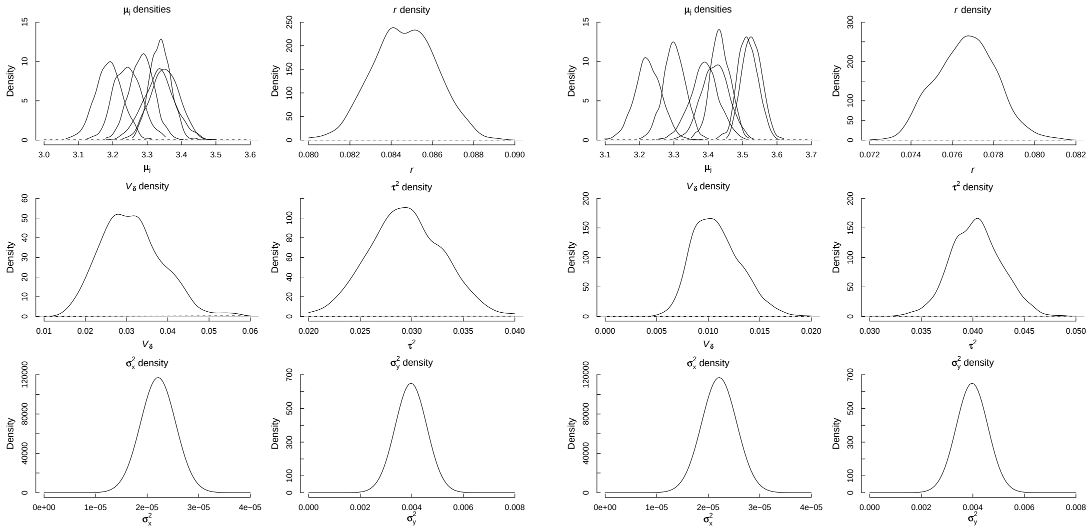
$$
\begin{gathered}
-2\bar{\sigma}_z \leq \epsilon_{it} \leq +2\bar{\sigma}_z \\
\exp(-2\bar{\sigma}_z) \leq \exp(\epsilon_{it}) \leq \exp(+2\bar{\sigma}_z) \\
100(\exp(-2\bar{\sigma}_z) - 1)(\%) \leq \text{measurement error}(\%) \leq 100(\exp(+2\bar{\sigma}_z) - 1)(\%)
\end{gathered}
$$

Results showed a very good estimation of the DBH with a low measurement error (0.93%). Height was also quite well estimated with an error close to 10%. The two other variables, crown height and crown radius, were quite difficult to measure in the field and had a range of precision of approximately 50% and 30%, respectively.

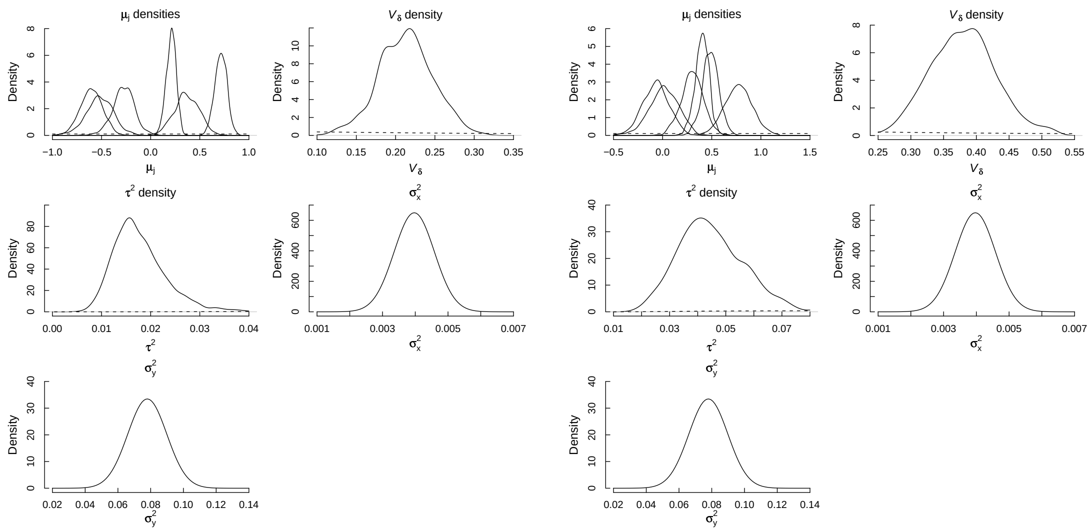
Variable	Model	Mean (σ^2_z)	Sd (σ^2_z)	Measurement error (%) confidence interval at 95%	
				lower bound	upper bound
DBH	$z_{it} = LN(\log z_{i,0}, \sigma_z^2)$	2.21E-05	3.42E-06	-0.93	0.94
H		3.97E-03	6.13E-04	-11.84	13.43
CH		7.78E-02	1.23E-02	-42.76	74.69
CR		2.42E-02	3.75E-03	-26.74	36.50

Appendix S4: **Means and standard deviations for variance associated to measurement errors**. Means and variances were calculated from the thousand simulations of σ_z^2 obtained with MCMC. Credible interval at 95% for the measurement errors were computed. As errors were multiplicative they were expressed in percentage.

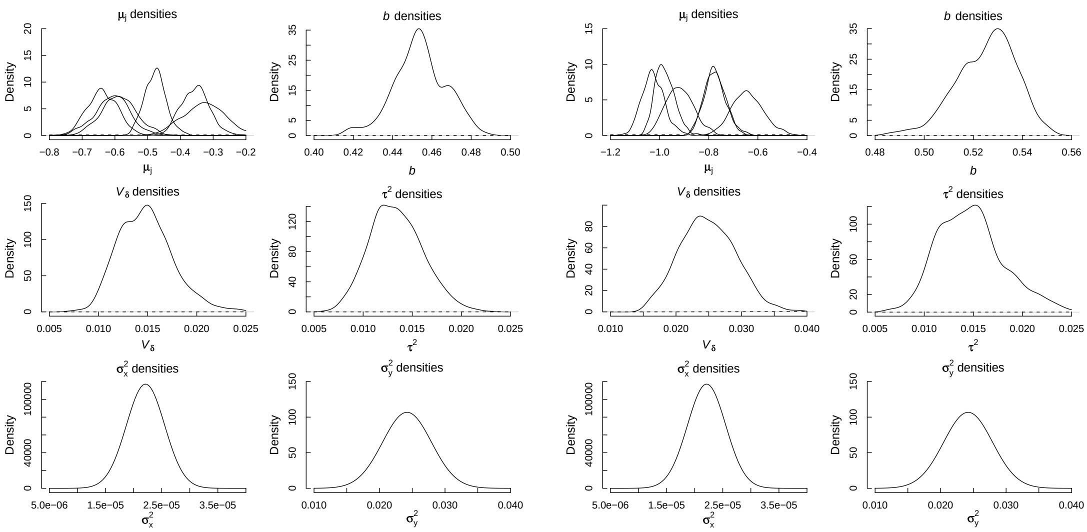
Abies alba *Picea abies*



(a)



(b)



(c)

Appendix S5: **Posteriors and priors for parameters**. Allometries are: (a) height as a function of DBH, (b) crown height as a function of height and (c) crown radius as a function of DBH. Priors are represented with dashed lines (- - -) and posteriors with plain lines (—). We used informative priors for the measurement error variance on response (σ_y^2) and on covariate (σ_x^2). All other priors were taken non-informative.

(a) H-DBH		*Abies alba*		*Picea abies*	
Parameter	Signification	Mean	Sd	Mean	Sd
μ *[Luan]*		3.34E+00	4.74E-02	3.42E+00	3.94E-02
μ *[Miroir1]*		3.24E+00	4.03E-02	–	–
μ *[Miroir3]*		3.35E+00	4.20E-02	–	–
μ *[Premol]*		3.19E+00	4.02E-02	3.39E+00	4.17E-02
μ *[Queige]*	site fixed effects	3.28E+00	3.59E-02	3.53E+00	2.84E-02
μ *[Sixt]*		–	–	3.52E+00	2.93E-02
μ *[SteFoy]*		–	–	3.44E+00	2.89E-02
μ *[StRhemy]*		–	–	3.30E+00	3.26E-02
μ *[Teppas]*		3.34E+00	3.15E-02	3.23E+00	3.96E-02
r	fixed f parameter	8.46E-02	1.55E-03	7.67E-02	1.45E-03
V	variance of individual random effects	3.12E-02	7.42E-03	1.08E-02	2.33E-03
$\sigma^2 y$	variance of log-measurement errors on response H	3.97E-03	6.14E-04	3.97E-03	6.14E-04
$\sigma^2 x$	variance of log-measurement errors on covariate DBH	2.21E-05	3.40E-06	2.21E-05	3.40E-06
τ^2	variance of log-errors	2.94E-02	3.61E-03	4.03E-02	2.45E-03

(b) CH-H		*Abies alba*		*Picea abies*	
Parameter	Signification	Mean	Sd	Mean	Sd
μ *[Luan]*		3.67E-01	1.26E-01	7.72E-01	1.39E-01
μ *[Miroir1]*		-6.04E-01	1.12E-01	–	–
μ *[Miroir3]*		-2.76E-01	1.11E-01	–	–
μ *[Premol]*	site fixed effects	-5.19E-01	1.38E-01	1.37E-02	1.46E-01
μ *[Queige]*		7.14E-01	6.16E-02	4.88E-01	8.45E-02
μ *[Sixt]*		–	–	-6.95E-02	1.33E-01
μ *[SteFoy]*		–	–	4.01E-01	6.99E-02
μ *[Teppas]*		2.09E-01	5.00E-02	3.03E-01	1.12E-01
V	variance of individual random effects	2.13E-01	3.44E-02	3.80E-01	5.10E-02
$\sigma^2 y$	variance of log-measurement errors on response CH	7.79E-02	1.19E-02	7.79E-02	1.19E-02
$\sigma^2 x$	variance of log-measurement errors on covariate H	3.97E-03	6.14E-04	3.97E-03	6.14E-04
τ^2	variance of log-errors	1.78E-02	5.36E-03	4.48E-02	1.16E-02

(c) CR-DBH		*Abies alba*		*Picea abies*	
Parameter	Signification	Mean	Sd	Mean	Sd
μ *[Luan]*		-3.27E-01	6.45E-02	-6.41E-01	6.46E-02
μ *[Miroir1]*		-6.38E-01	4.57E-02	–	–
μ *[Miroir3]*		-5.88E-01	5.26E-02	–	–
μ *[Premol]*	site fixed effects	-6.07E-01	5.51E-02	-9.18E-01	5.55E-02
μ *[Queige]*		-3.54E-01	4.30E-02	-7.74E-01	4.48E-02
μ *[Sixt]*		–	–	-1.03E+00	4.56E-02
μ *[SteFoy]*		–	–	-9.78E-01	3.99E-02
μ *[Teppas]*		-4.77E-01	3.26E-02	-7.74E-01	4.19E-02
b	fixed f parameter	4.54E-01	1.33E-02	5.25E-01	1.20E-02
V	variance of individual random effects	1.48E-02	2.78E-03	2.49E-02	4.23E-03
$\sigma^2 y$	variance of log-measurement errors on response CR	2.42E-02	3.73E-03	2.42E-02	3.73E-03
$\sigma^2 x$	variance of log-measurement errors on covariate DBH	2.21E-05	3.40E-06	2.21E-05	3.40E-06
τ^2	variance of log-errors	1.34E-02	2.70E-03	1.48E-02	3.35E-03

Appendix S6: **Means and standard deviations of the estimated parameters for the best allometric models**. Allometries are: (a) height as a function of DBH, (b) crown height as a function of height and (c) crown radius as a function of DBH.